are too small to permit a clear decision between the methods, especially in view of the unknown effect of such substituents as the acetyl group in the steroids.[11] However, these findings do clearly indicate that the anomalous deshielding of the C-18 hydrogens of substituted 14α,17α-etheno-15-pregnen-20-ones cannot be attributed solely to the presence of the 15,16 double bond.[12]

Experimental Section

Melting points were determined in open capillary tubes on a Mel-Temp apparatus and are uncorrected. Elemental analyses were performed by Galbraith Microanalytical Laboratories, Knoxville, Tenn. The ir spectra were determined on a Beckman IR-8 spectrophotometer. Nmr spectra were run in $CDCl_3$ on a Varian A-60 spectrometer and are reported in parts per million downfield from a TMS standard.

3β-Acetoxy-14α,17α-ethano-5-pregnen-20-one.—A solution of 120 mg of **1** in 110 ml of ethanol and 5 ml of H_2O was hydrogenated over 30 mg of 10% Pd/C at an initial pressure of 3.16 kg/cm^2 for 16 hr. Standard work-up afforded 3β-acetoxy-14α,17α-ethano-5-pregnen-20-one in a yield of 68 mg as colorless needles (from ethanol): mp 140–141°; ν^{Nujol} 1739, 1701, 1239 cm^{-1}. The nmr spectrum had peaks at δ 0.90 (s, C-18 H's), 1.04 (s, C-19 H's), 2.02 [s, $OC(=O)CH_3$], 2.10 (s, C-21 H's), and 5.40 (m, C-6 vinyl H).

Anal. Calcd for $C_{25}H_{26}O_3$: C, 78.08; H, 9.44. Found: C, 78.12; H, 9.50.

3β-Acetoxy-16α-carbomethoxyamido-14α,17α-ethano-5-pregnen-20-one (**5**).—A solution of 1.00 g of **4** in 75 ml of acetone was cooled in an ice bath, and a solution of 0.41 g of triethylamine in 25 ml of acetone was added to it dropwise, with stirring. The resulting solution was stirred for 0.5 hr, and then 0.43 g of ethyl chloroformate in 25 ml of acetone was added dropwise. Stirring was then continued for 1.5 hr. A solution of 0.34 g of NaN_3 in 1 ml of water was then poured into the cold solution. The resulting mixture was stirred for 3 hr and was then concentrated under vacuum. The residue was partitioned between ether and water. The organic layer was dried, filtered, and concentrated. The residue was dissolved in 65 ml of benzene and heated under reflux for 1 hr. After 15 ml of absolute methanol had been added to the hot solution, reflux was continued overnight. The resulting solution was concentrated. The residue crystallized from ethanol to afford **5** in a yield of 0.89 g (66%), as colorless prisms: mp 225–226°; ν^{CHCl_3} 3468, 1720, 1512, 1250–1210, and 1030 cm^{-1}. The nmr spectrum had singlets at δ 1.05 (C-18 and 19 H's), 2.04 [$OC(=O)CH_3$], 2.18 (C-21 H's), 3.67 (OCH_3), and a multiplet at 5.38 (C-6 H).

Anal. Calcd for $C_{27}H_{29}NO_5$: C, 70.87; H, 8.59; N, 3.06. Found: C, 70.98; H, 8.48; N, 2.97.

3β-Acetoxy-14α,17α-ethano-5,15-pregnadien-20-one (**2**).—A mixture of 475 mg of **5** and 107 mg of NaOAc in 10 ml of ether was stirred and cooled in a Dry Ice–acetone bath while approximately 0.2 ml of N_2O_4 (purified by passage through P_2O_5) was added. Stirring was continued for 1 hr in the cold and then for 10 min at 0°. The inorganic salt was removed by filtration. The filtrate was extracted with 5% aqueous $NaHCO_3$ and then was washed with water. The neutral solution was dried ($MgSO_4$), filtered, and concentrated. The residue, on tlc, appeared to consist of starting material and of two faster moving spots of very similar R_f. The mixture was purified by thick layer chromatography on silica gel. The plates were developed three times with 15% ethyl acetate–85% hexane, and the fast-moving zone was rechromatographed under similar conditions. The fastest moving band gave **2** in a yield of 23 mg, as fine colorless needles from MeOH: mp 120–121°; ν^{CHCl_3} 1725, 1691, 1252, and 1025 cm^{-1}. The nmr spectrum had singlets at δ 0.89 (C-18 H's), 1.04 (C-19 H's), 2.03 [$OC(=O)CH_3$], and 2.17 (C-21 H's) and peaks corresponding to vinyl hydrogens at δ 5.45 (C-6 H, m), 6.02 (C-15 H, d, J = 6 Hz), and 6.18 (C-16 H, d, J = 6 Hz).

Anal. Calcd for $C_{25}H_{34}O_3$: C, 78.49; H, 8.96. Found: C, 78.68; H, 9.06.

3β-Acetoxy-14α,17α-ethano-16,16′-cyclo-5-pregnen-20-one (**7**). **A.**—Extraction of the second fastest moving band on the tlc of the reaction mixture, which gave **2**, afforded **7**, in a yield of 50 mg, as colorless prisms from MeOH: mp 190–190.5°; ν^{CHCl_3} 1723, 1666, 1253, and 1025 cm^{-1}. The nmr spectrum had singlets at δ 1.01 (C-18 H's), 1.04 (C-19 H's), 1.90 (C-21 H's shielded by cyclopropane), and 2.03 [$OC(=O)(CH_3)$], and a multiplet corresponding to the C-6 vinyl hydrogen at δ 5.38.

Anal. Calcd for $C_{25}H_{34}O_3$: C, 78.49; H, 8.96. Found: C, 78.51; H, 9.12.

B.—During the work-up of the mixture resulting from Hunsdiecker reaction of **4**, the crude product was chromatographed over alumina and benzene eluted **3**. Further development of the column with 1:1 benzene–ethyl acetate resulted in the elution of a fraction which crystallized from ethanol to give 143 mg (31%) of impure **7** as white crystals, mp 189.5–190.5°.

Anal. Calcd for $C_{25}H_{34}O_3$: C, 78.49; H, 8.96; mol wt, 382.5. Found: C, 76.45, 76.64, 77.38; H, 8.92, 8.80, 8.74; mol wt, 390.

C.—Acetylation of **8**, by the usual method, afforded **7** which crystallized from ethanol as colorless needles identical in melting points, mixture melting point, ir, and nmr with **7** prepared as in A.

3β-Hydroxy-14α,17α-ethano-16,16′-cyclo-5-pregnen-20-one (**8**).—A mixture of 158 mg of **7** (prepared as in B), 300 mg of KOH, 25 ml of MeOH, and 3 ml of H_2O was stirred at room temperature for 48 hr and then under reflux for 6 hr. The mixture was cooled to room temperature and then concentrated under reduced pressure. The residue was partitioned between ether and H_2O. The organic layer was dried ($MgSO_4$), filtered, and concentrated. The residue crystallized from MeOH to afford **8** as tiny colorless rods, in a yield of 125 mg: mp 183–185°; ν^{Nujol} 3600, 1662 cm^{-1}. The nmr spectrum had singlets at δ 0.98 (C-18 H's), 1.03 (C-19 H's), and 1.87 (C-21 H's shielded by cyclopropane) and a multiplet at δ 5.39 (C-6 H).

Anal. Calcd for $C_{23}H_{32}O_2 \cdot H_2O$: C, 77.05; H, 9.56. Found: C, 77.20; H, 9.61.

Later, 44 mg of **8** was purified by tlc on silica gel. The plate was developed with 20% EtOAc in benzene. The sample isolated (34 mg) was dissolved in ethanol containing 5% benzene and evaporated to dryness under vacuum. This process was then repeated six times. The residue was dried for 3 days at 100° over P_2O_5.

Anal. Calcd for $C_{23}H_{32}O_2$: C, 81.13; H, 9.47. Found: C, 79.35; H, 9.53.

Registry No.—**1**, 19605-66-4; **2**, 35639-00-0; **5**, 35639-01-1; **7**, 35639-02-2; **8**, 35639-03-3.

Acknowledgment.—This work was supported, in part, by research grants AM-006900 from the National Institute of Arthritis and Metabolic Diseases and by general research grant 5 S01 FR-05454 from the U. S. Public Health Service.

---

[11] N. Bhacca and D. Williams, "Applications of NMR Spectroscopy in Organic Chemistry," Holden-Day, San Francisco, Calif., 1964, p 29.

[12] The methyl hydrogens of 7-methylbicyclo[2.2.1]heptene have been reported to appear in the nmr spectrum (CCl_4) at δ 0.70 for the cis compound and at δ 0.79 (CCl_4) for the trans.[13]

[13] Reference 8, p 84.

A Facile Synthesis of (±)-*ar*-Artemisene *via* Olefin Metalation

Robert J. Crawford

The Procter & Gamble Company, Miami Valley Laboratories,
Cincinnati, Ohio 45239

Received May 2, 1972

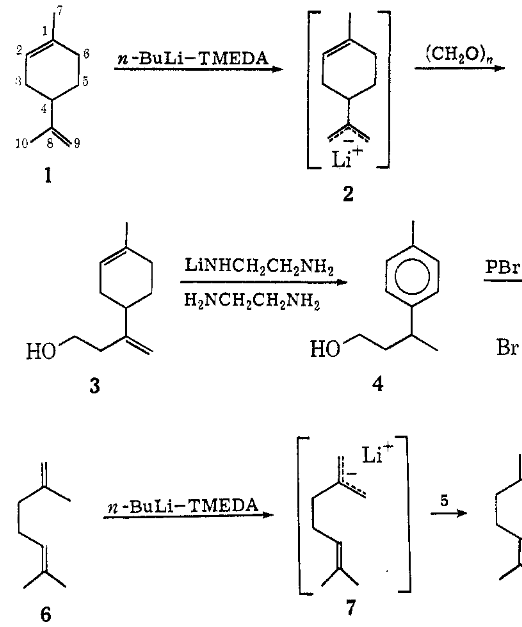
The monocyclic diterpene hydrocarbon *ar*-artemisene (**8**) was isolated from wormwood oil by Šorm and coworkers in 1951.[1] Its racemate was later synthesized

---

[1] F. Šorm, M. Suchý, F. Vonášek, J. Plíva, and V. Herout, *Collect. Czech. Chem. Commun.*, **16**, 268 (1951); *Chem. Listy*, **45**, 135 (1951).

by Vig, *et al.*,[2] by three different routes. Although these syntheses are unambiguous and serve to confirm the structure of **8**, they require a large number of steps from readily available materials. This communication reports a simple four-step synthesis of (±)-*ar*-artemisene starting with the hydrocarbons limonene (**1**) and geraniolene (**6**) (Chart I). Two of the steps make use of the olefin metalation process that we have described in detail in the case of limonene.[3]

CHART I



We have shown that limonene, on reaction with the 1:1 complex of *n*-butyllithium and *N*,*N*,*N*′,*N*′-tetramethylethylenediamine (TMEDA), undergoes selective metalation at C-10 to afford the 2-substituted allyllithium species **2**.[3] Treatment of this intermediate with various reagents provides a general synthesis of 10-substituted limonene derivatives, and the products are obtained in high purity after simple distillation. We have now found that geraniolene can be converted in a similar manner to products derived from the intermediate **7**. However, the reaction mixtures from metalation–derivatization of geraniolene usually contain a complex mixture of minor components that must be separated from the major product by preparative glpc.[4]

In the present study, paraformaldehyde was added to a solution of **2** to provide the alcohol **3** in 57% yield.[5] Treatment of **3** with *N*-lithioethylenediamine in refluxing ethylenediamine[7] resulted in smooth conversion to the aromatic alcohol **4** in 72% yield. The bromide **5** was obtained from **4** by a known procedure[8] using phosphorus tribromide. Geraniolene was metalated with *n*-butyllithium–TMEDA under conditions similar to those used for limonene. Addition of the bromide **5** to the solution containing **7** afforded a mixture in which **8** was the major component. The (±)-*ar*-artemisene was purified by preparative glpc and was identified by its spectral properties and by comparison of its ir spectrum with that reported[1,2] for the natural product.

Experimental Section

Boiling points are uncorrected. Infrared spectra were recorded on a Perkin-Elmer Model 257 grating spectrophotometer. Nuclear magnetic resonance spectra were determined on a Varian Associates Model HA-100 spectrometer. Chemical shifts are reported in parts per million on the τ scale, with tetramethylsilane as internal standard; coupling constants are in hertz. Nmr data are recorded in the following order: chemical shift, multiplicity (where s = singlet, d = doublet, t = triplet, q = quartet, m = multiplet), integration, coupling constant, interpretation. Samples were run in 1.5-mm-o.d. glass capillary tubes according to the micro-nmr technique developed in these laboratories.[9] Mass spectra were determined on an Atlas CH-4 spectrometer. Gas chromatographic separations were carried out on a Varian Aerograph Model 202-2B instrument equipped with the following columns: column A, 10 ft × 0.25 in. stainless steel packed with 20% SE-30 on 60/80 mesh AW-DMCS Chromosorb W; column B, 10 ft × 0.25 in. stainless steel packed with 20% FFAP on 60/80 mesh AW-DMCS Chromosorb W; column C, 20 ft × 0.375 in. stainless steel packed with 20% Carbowax 20 M on 45/60 mesh AW-DMCS Chromosorb W. Helium was used as the carrier gas, and was operated at a flow rate of 60 ml/min on columns A and B and 150 ml/min on column C. All reactions were run under a static atmosphere of dry argon using an apparatus of the type described by Johnson and Schneider.[10] The procedure for isolation of products following solvent extraction consisted of drying the organic solution over anhydrous sodium sulfate and removal of solvent at reduced pressure on a rotary evaporator. Molecular distillations were carried out in a vertical bulb-to-bulb apparatus. Microanalysis was performed by Scandinavian Microanalytical Laboratory, Herlev, Denmark.

Materials.—*d*-Limonene was obtained from Matheson Coleman and Bell. Geraniolene was prepared in 95% yield by pyrolytic distillation of geranic acid (Fritzsche Bros.) according to the procedure of Bateman, *et al.*[11] *n*-Butyllithium in hexane was supplied by Foote Mineral Co. *N*,*N*,*N*′,*N*′-Tetramethylethylenediamine (TMEDA) and ethylenediamine were both obtained from Matheson Coleman and Bell and were dried immediately prior to use by distillation from calcium hydride.

3-(4-Methyl-3-cyclohexen-1-yl)-3-buten-1-ol (3).—To a stirred solution of 50.0 ml (0.075 mol) of 1.5 *M* *n*-butyllithium in hexane was added dropwise 11.3 ml (8.8 g, 0.076 mol) of dry TMEDA, followed by 25.0 ml (21.0 g, 0.154 mol) of *d*-limonene. The resulting mixture was stirred for 1 hr and allowed to stand overnight at room temperature. Stirring was resumed, and 3.0 g (0.10 mol) of paraformaldehyde was added in small portions over a period of 30 min. The solution temperature was maintained below 30° during the addition and for a period of 3.5 hr after the addition was complete. Water (50 ml) was added, the layers were separated, and the aqueous solution was extracted with three portions of ether. The combined organic solutions were washed successively with 5% sodium chloride, 1 *M* hydrochloric acid, and 5% sodium chloride solutions, and were dried and evaporated.

---

[2] (a) O. P. Vig, K. L. Matta, A. Singh, and I. Raj, *J. Indian Chem. Soc.*, **42**, 773 (1965); (b) O. P. Vig, R. Anand, B. Ram, and B. Vig, *Indian J. Chem.*, **5**, 538 (1967); (c) O. P. Vig, J. M. Sehgal, M. M. Mahajan, and S. D. Sharma, *J. Indian Chem. Soc.*, **46**, 887 (1969).

[3] R. J. Crawford, W. F. Erman, and C. D. Broaddus, *J. Amer. Chem. Soc.*, **94**, 4298 (1972).

[4] Although minor products have not been identified, many of them probably arise from competing reactions in the metalation step (*e.g.*, metalation at the trisubstituted double bond or double bond migration followed by reactions of conjugated dienes). Side reactions are virtually insignificant in the metalation of **1**.

[5] This alcohol can also be obtained in comparable yield by acid-catalyzed addition of formaldehyde to limonene.[6] The metalation route offers the advantage that the product is obtained in higher purity.

[6] (a) A. T. Blomquist and R. J. Himics, *J. Org. Chem.*, **33**, 1156 (1968); (b) R. J. Himics, Ph.D. Thesis, Cornell University, Ithaca, N. Y., 1967.

[7] (a) B. N. Joshi, R. Seshadri, K. K. Chakravarti, and S. C. Bhattacharyya, *Tetrahedron*, **20**, 2911 (1964); (b) B. S. Tyagi, B. B. Ghatge, and S. C. Bhattacharyya, *ibid.*, **19**, 1189 (1963); (c) B. S. Tyagi, B. B. Ghatge, and S. C. Bhattacharyya, *J. Org. Chem.*, **27**, 1430 (1962); (d) L. Reggel, S. Friedman, and I. Wender, *ibid.*, **23**, 1136 (1958).

[8] V. K. Honwad and A. S. Rao, *Tetrahedron*, **21**, 2593 (1965).

[9] L. V. Haynes and C. D. Sazavsky, *Appl. Spectrosc.*, **24**, 452 (1970).

[10] W. S. Johnson and W. P. Schneider, "Organic Syntheses," Collect. Vol. IV, Wiley, New York, N. Y., 1963, p 132.

[11] L. Bateman, J. I. Cunneen, and E. S. Waight, *J. Chem. Soc.*, 1714 (1952).

Distillation of the residue afforded 12.5 g of recovered limonene and 5.89 g (57% based on limonene consumed) of **3** as a colorless oil, bp 72–84° (0.1–0.15 mm) [lit.[6] bp 80–83° (0.2–0.3 mm)], glpc purity 94% (column B, 225°); the ir and nmr spectra were identical with the published spectra of **3**.[6]

3-*p*-Tolyl-1-butanol (**4**).—To 30 ml of vigorously stirred, anhydrous ethylenediamine at 100–110° was added 0.417 g (0.06 g-atom) of lithium wire in small pieces over a period of 30 min. When the blue color of the solution had disappeared, the mixture was heated to reflux, and a solution of 1.0 g (0.006 mol) of **3** in 5 ml of ethylenediamine was added. The solution was refluxed for 45 min, and then was cooled in ice while 70 ml of water was added. The resulting mixture was extracted with three portions of ether, and the combined extracts were washed with 5% sodium chloride solution, dried, and evaporated to afford 0.90 g of oil. Molecular distillation [bath temperature >60° (0.1 mm)] of this material afforded 0.71 g (72%) of **4** as a colorless oil: glpc purity 94% (column A, 200°); the ir spectrum was in close agreement with published ir data for **4**;[12] nmr ($CDCl_3$) τ 3.02 (s, 4 H, *p*-$CH_3C_6\mathbf{H}_4$–), 6.60 (t, 2 H, J = 7 Hz, –$\mathbf{CH}_2OH$), 7.12 (s, 1 H, –OH), 7.25 (m, 1 H, $C_7H_7\mathbf{CH}CH_3$–), 7.78 (s, 3 H, *p*-$\mathbf{CH}_3C_6H_4$–), 8.27 (q, 2 H, J = 7 Hz, –$\mathbf{CH}_2CH_2OH$), and 8.81 (d, 3 H, J = 7 Hz, $C_7H_7CH\mathbf{CH}_3$–).

1-Bromo-3-*p*-tolylbutane (**5**).—To 0.38 ml (4.0 mmol) of freshly distilled phosphorus tribromide cooled to 0° was added a solution of 0.66 g (4.0 mmol) of **5** in 2 ml of anhydrous petroleum ether. The mixture was stirred at room temperature for 18 hr, and then was poured into 10 ml of cold water. The resulting mixture was extracted with four portions of ether, and the combined extracts were washed twice with water and then dried and evaporated to afford 0.80 g of oil. Molecular distillation [bath temperature >80° (0.07 mm)] of this material afforded 0.38 g (42%) of **5** as a pale yellow oil: glpc purity 92% (column A, 200°); the ir spectrum was in close agreement with published ir data for **5**;[3] nmr ($CDCl_3$) τ 3.00 (s, 4 H, *p*-$CH_3C_6\mathbf{H}_4$–), 6.86 (m, 2 H, –$\mathbf{CH}_2Br$), 7.16 (m, 1 H, $C_7H_7\mathbf{CH}CH_3$–), 7.77 (s, 3 H, *p*-$\mathbf{CH}_3C_6H_4$–), 8.00 (m, 2 H, –$\mathbf{CH}_2CH_2Br$), and 8.80 (d, 3 H, J = 7 Hz, $C_7H_7CH\mathbf{CH}_3$–).

(±)-*ar*-**Artemisene** (**2-Methyl-6-methylene-10-*p*-tolyl-2-undecene**) (**8**).—To a stirred solution of 5.0 ml (7.5 mmol) of 1.5 *M* *n*-butyllithium in hexane was added 0.87 g (7.5 mmol) of dry TMEDA followed by 1.86 g (15.0 mmol) of geraniolene (**6**). The resulting mixture was stirred for 3 hr at room temperature, after which a 2.3-ml aliquot was removed, placed in a separate flask, and cooled below 0°. To this aliquot was added dropwise 0.354 g (1.56 mmol) of **5**. The reaction mixture was allowed to warm to room temperature over a period of 45 min, diluted with water, and extracted with three portions of ether. The combined extracts were washed successively with 5% sodium chloride, 1 *M* hydrochloric acid, 5% sodium chloride, 2% sodium bicarbonate, and 5% sodium chloride solutions, and were dried and evaporated. Molecular distillation of the residue afforded 0.245 g (58%) of colorless oil. Glpc analysis of this material (column C, 240°) showed a mixture in which the component with longest retention time (65 min) was a single major constituent representing *ca.* 50% of the total. This substance was purified by preparative glpc (column C, 240°) followed by molecular distillation to afford (±)-*ar*-artemisene as a colorless oil: bp (bath) 110° (0.03 mm) [lit.[2b] bp 110° (4–5 mm)]; glpc purity 100% (column B, 240°); ir (film) 3075 (>C═CH_2), 3050 and 3020 (–C_6H_4–), 3010 (>C═CH–), 1649 (>C═CH_2), 1519 (–C_6H_4–), 1456, 1380, 1311 (w), 1110, 1044 (w), 1026, 990 (w), 894 (>C═CH_2), 822 (*p*-C_6H_4–), and 730 cm^{-1} (this spectrum agrees closely with published ir data for **8**[1,2]); nmr ($CDCl_3$) τ 3.00 (s, 4 H, *p*-$CH_3C_6\mathbf{H}_4$–), 4.94 [m, 1 H, –$\mathbf{CH}$═$C(CH_3)_2$], 5.36 (broadened s, 2 H, >C═CH_2), 7.39 (m, 1 H, $C_7H_7\mathbf{CH}CH_3$–), 7.75 (s, 3 H, *p*-$\mathbf{CH}_3C_6H_4$–), 7.9–8.2 (m, 6 H, allylic –CH_2–), 8.3–8.8 [m, 10 H, including 8.34 and 8.43 (two broadened s, –CH═$C(\mathbf{CH}_3)_2$)], and 8.80 (d, 3 H, J = 7 Hz, $C_7H_7CH\mathbf{CH}_3$–); mass spectrum (70 eV) *m/e* (rel intensity) 270 (7), 255 (1), 227 (13), 201 (1), 199 (2), 185 (6), 171 (4), 159 (12), 157 (7), 145 (26), 132 (65), 131 (21), 119 (100), 109 (35), 105 (25), 91 (19), 69 (74), 55 (12).

Anal. Calcd for $C_{20}H_{30}$: C, 88.82; H, 11.18. Found: C, 89.01; H, 11.20.

Registry No.—**8**, 19907-39-2.

Acknowledgment.—The technical assistance of Mr. F. W. Vonderahe is gratefully acknowledged.

---

[12] V. K. Honwad and A. S. Rao, *Tetrahedron*, **20**, 2921 (1964).

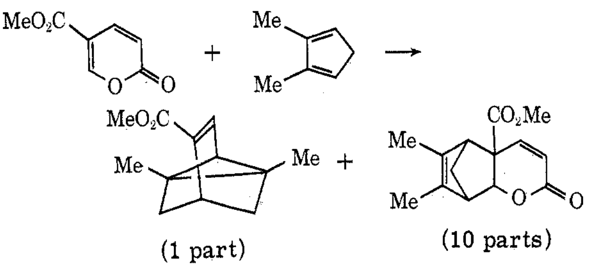
The Diels–Alder Dimerization of 2-Pyrone

DAVID L. WHITE AND DIETMAR SEYFERTH*

Department of Chemistry, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

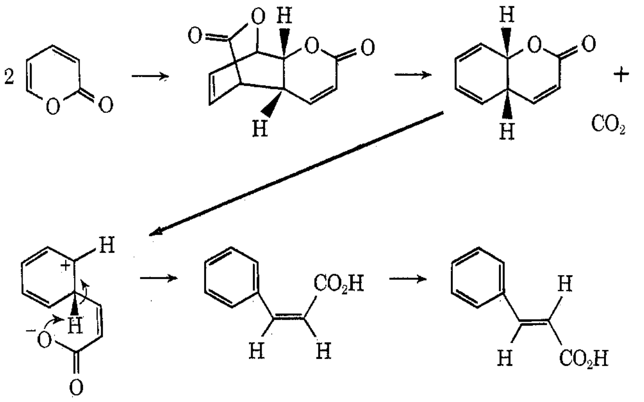
Received April 24, 1972

The recent report by Imagawa, *et al.*,[1] concerning reactions in which a 2-pyrone (methyl coumalate) functions as a dienophile (rather than as a diene) in a Diels–Alder reaction (eq 1) prompts us to relate an observation made during the course of our studies of the Diels–Alder reactions between 2-pyrone and group IVB element substituted acetylenes.[2,3] Although the reaction of 2-pyrone proceeded well with acetylenes such as $Me_3SiC{\equiv}CSiMe_3$,[3] $Me_3GeC{\equiv}CGeMe_3$,[3] and $Me_3SnC{\equiv}CSnMe_3$,[2] the expected adduct, 1,2-di-*tert*-butylbenzene, could not be obtained on attempted reaction with di-*tert*-butylacetylene, even after solutions of the reactants in bromobenzene had been heated in a sealed tube at 210° for 5 days. However, a white solid, subsequently identified as *trans*-cinnamic acid by mixture melting point and comparison of its ir and nmr spectra with those of an authentic sample, was isolated from such a reaction mixture in low yield. The formation of this unexpected product was explained[4] in terms of a Diels–Alder reaction of 2-pyrone with itself, the dienophilic C═C bond being the 5,6 double bond as in the examples of the Japanese workers (Scheme I).



(1)

SCHEME I



The strenuous reaction conditions would serve to explain the degradative process outlined. Such Diels–

---

[1] T. Imagawa, N. Sueda and M. Kawanisi, *J. Chem. Soc., Chem. Comm.*, 388 (1972).

[2] A. B. Evnin and D. Seyferth, *J. Amer. Chem. Soc.*, **89**, 952 (1967).

[3] D. Seyferth and D. L. White, *J. Organometal. Chem.*, **34**, 119 (1972).

[4] D. L. White, Ph.D. Thesis, Massachusetts Institute of Technology, 1970.